Search and Detection
Continuous Search

1. Assume a continuous search with constant detection rate γ=.15 det/minute.

a. What is the probability that 1 or more detections occur during the first 3 minutes?
b. What is the probability of initial detection during the first 3 minutes?
c. What is the expected number of detections during the first 3 minutes?
d. What is the probability that 1 or more detections occur during the time interval 2 to 4 minutes?
e. What is the probability of initial detection occuring during the time interval 2 to 4 minutes?
f. What is the expected number of detections during the time interval 2 to 4 minutes?
g. What is the mean time to detection given detection?
h. What is the mean time to detection?

2. (Washburn, S&D p.2-16 #2) Consider random search which terminates either at detection or after τ time units, which ever comes first. What is the expected time spent searching?

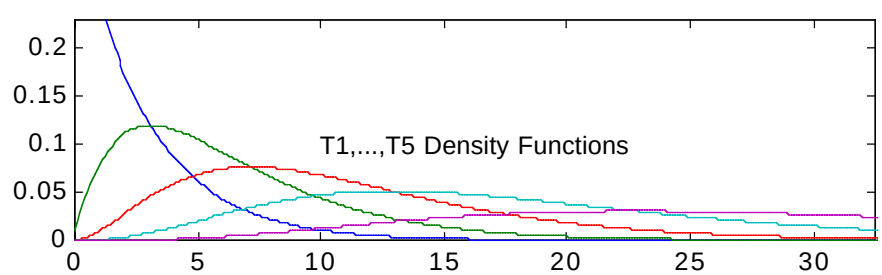
3. For an inverse cube law sensor, the lateral range curve is given by

$$l(x) = 1 - \exp\left[\frac{-2kAh}{v(h^2 + x^2)}\right].$$

a. For a fixed lateral range x, find the height h*(x) which maximizes l(x). (This is the best height to fly when searching with an inverse cube law sensor for an object at distance x off the track.)
b. For k=10,000 hr^{-1}, A=.01 nm^2, and v=200 kt, calculate and plot a lateral range curve l(x) for each height h=.25:.25:6 nm.
c. For each of the heights h in b., calculate sweep width, W(h). Plot W(h) vs. h. What value h* maximizes W(h)?

4. Let searcher speed v = 400 km/hr; searcher cookie-cutter detector range R = 10 km; search area size A = 200 km x 200 km; and the cookie-cutter sensor detects targets within R with probability .7. There are 5 idential targets to be detected, and each target is available for detection 50% of the time.

a. What are the random search detection rates when 5, 4, 3, 2, and 1 targets remain to be detected? (Ans: 5g, 4g, 3g, 2g, g with g=.07 hr^{-1})

b. Using random search, what are the mean times to detect the first target, second target,..., fifth target? (Ans: 2.86, 6.43, 11.19, 18.33, 32.62 hr.)
Hint: If Ti is the time of initial detection for target i, then
T1 ~ exp(5g), (T2-T1) ~ exp(4g), ... , (T5-T4) ~ exp(g). So,
E[T1] = 1/(5g);
E[T2] = E[T1 + (T2-T1)] = E[T1] + E[T2-T1] = E[T1] + 1/(4g);
E[T3] = E[T1 + (T2-T1) + (T3-T2)] = E[T1]+E[T2-T1]+E[T3-T2] = E[T2] + 1/(3g); etc.

c. Use MATLAB to compute and plot the density functions for T1, T2,..., T5.
Hint: Use a discrete approximation of the continuous problem. If Xi is a discrete random variable with probabilities mi, then the sum (X1+X2) is a discrete random variable with probabilities given by the convolution of X1 and X2. Using MATLAB, CONV(m1,m2) gives these discrete probabilities.