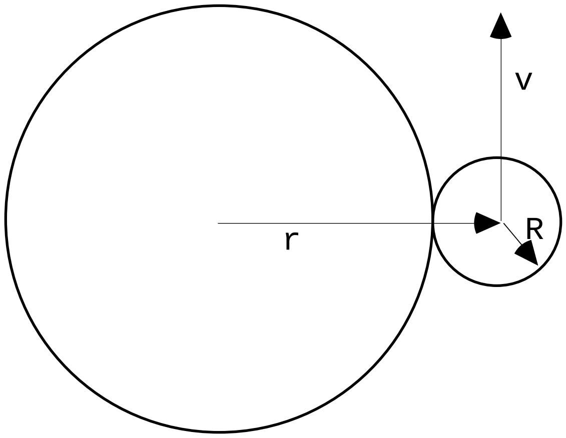
Trapping Sprials

S&D 1-16 to 1-21

1. <u>Maximum Trapping Radius</u>. If the searcher with speed v and detection range R can complete a circle of radius r in less than or equal to the time required for a target with speed u to cover a distance of 2R, then the target is trapped inside a circle of radius (r-R).



Conditions for Trap:

$$\frac{2\pi r}{v} \le \frac{2R}{u}.$$

So the maximum trapping radius is

$$r_{\max} = \frac{Rv}{\pi u}.$$

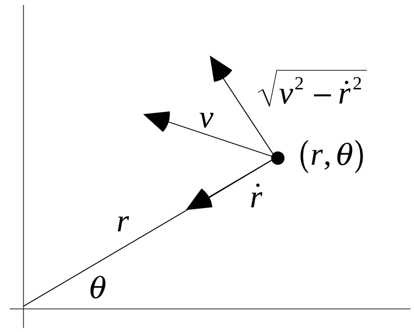
Also, if $r_{init} < r_{max}$, then it is possible to sprial in and eventually detect all targets within (r_{init}-R) of datum.

2. <u>Computing the Trapping Sprial Track</u>. Let (r(t), θ(t)) be the searcher's position at time t. When the target is "just trapped", the target's maximum possible speed relative to the searcher (u - dr(t)/dt) must allow a distance of exactly 2R to be covered in the time required for the searcher to complete one revolution. (Note that for a collapsing spiral, dr(t)/dt is negative.) So,

$$\dot{r}(t) = u - 2R \Big/ \left(2\pi\, r(t) \big/ v\right) = u - \frac{Rv}{\pi r(t)}$$

$$\dot{\theta}(t) = \sqrt{v^2 - \dot{r}(t)^2}\ \Big/\ r(t)$$

Boundary conditions are $r(t_0) = r_{init}$ and $\theta(t_0) = \theta_{init}$.



3. <u>Time to Trap</u>. The time required to close from r_{init} (< r_{max}) to R is

$$\left(1/u\right)\left(r_{\max} \ln\left(\frac{r_{\max} - R}{r_{\max} - r_{\text{init}}}\right) - r_{\text{init}} + R\right).$$

Search of the Furthest-On-Disk (Archimedes Spiral)

1. Archimedes Spiral: $r(\theta) = S\theta/(2\pi)$. Note that r increases by track spacing S with each revolution.

2. This search is used when:
   - Searcher arrives too late to trap.
   - Desire is to search the furthest-on-DISK as uniformly as possible (as opposed to searching just the furthest-on-CIRCLE).

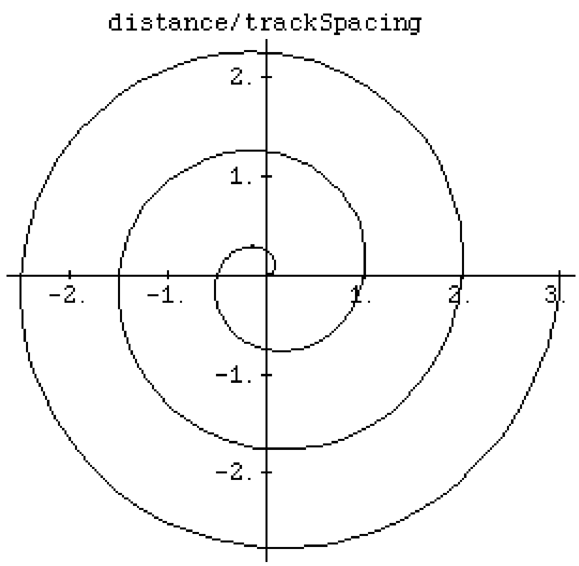
3. Searcher track starts at datum at time t=0.

4. Track spacing to approximate a uniform covering of the expanding FOD is $S^* = (4\pi u/v)*(u\tau - R)$. And the time required for the sensor to reach the FOC when using track spacing S* is $t^* = \tau - R/u$.

5. Approximate expressions for r(t) and $\theta(t)$:

$$r(t) \approx \sqrt{Svt/\pi}$$

$$\theta(t) = 2\pi r(t)/S \approx 2\sqrt{v\pi t/S}$$



Search of Furthest-On-Circle (Log Spiral)

1. For the Log Spiral,

$$r(\theta) = a_1 \exp(a_2 \theta) = u\tau \exp\left(\theta \Big/ \sqrt{(v/u)^2 - 1}\right),$$

so r increases by a multiplicative factor with each revolution.

2. This search keeps the searcher on the furthest-on-circle. The searcher's radial speed is always target speed u. So,

$$r(t) = u(t + \tau).$$

Searcher track starts at t=0, r=uτ and θ =0.

3. It is called a "log spiral" because for t≥0,

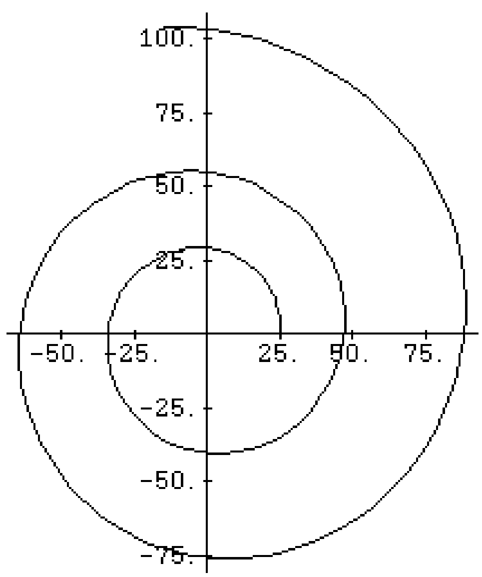
$$\theta(t) = \ln\left(\tfrac{t+\tau}{\tau}\right) \sqrt{(v/u)^2 - 1}.$$

Note: Time t =-τ is when the target starts fleeing. The searcher arrives on station and begins the search at time t=0.

4. Time required to search the FOC the first time is

$$t(2\pi) = \tau\left(\exp\left[2\pi \Big/ \sqrt{(v/u)^2 - 1}\right] - 1\right).$$

5. Sample track: u = 20kt, v = 200kt and τ =(25/20) hr.



The FOC is searched just over 2.25 times in this example.